Punkteverteilung:

1	2	3	4	Σ

Aufgabe (1)

Induktionsvorraussetzung

$\sum_{i=1}^{n}(-1)^{i+1}i = \frac{1+(-1)^{n+1}\cdot(2n+1)}{4}$

Induktionsanfang $n_0 = 1$

$\sum_{i=1}^{1}(-1)^{i+1}i = (-1)^{1+1}\cdot 1 = (-1)^2 \cdot 1 = 1$

$\frac{1+(-1)^(1+1)\cdot(2+1)}{4} = \frac{1+(-1)^2\cdot 3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1$

Induktionsschluss

$$\begin{aligned}
\sum_{i=1}^{n+1}(-1)^{i+1}i = & \left(\sum_{i=1}^{n}(-1)^{i+1}i\right) + (-1)^{n+1+1}\cdot(n+1) = \\
& \left(\sum_{i=1}^{n}(-1)^{i+1}i\right) + (-1)^{n+2}\cdot(n+1) = \\
& \frac{1+(-1)^{n+1}\cdot(2n+1)}{4} + (-1)^{n+2}\cdot(n+1) = \\
& \frac{1+(-1)^{n+1}\cdot(2n+1)}{4} + \frac{4(-1)^{n+2}\cdot(n+1)}{4} = \\
& \frac{1+(-1)^{n+1}\cdot(2n+1)+4(-1)^{n+2}\cdot(n+1)}{4} = \\
& \frac{1+(-1)^{n+1}\cdot[(2n+1)+4(-1)^{-1}\cdot(n+1)]}{4} = \\
& \frac{1+(-1)^{n+1}\cdot[(2n+1)+(-4)\cdot(n+1)]}{4} = \\
& \frac{1+(-1)^{n+1}\cdot[2n+1+(-4n-4)]}{4} = \\
& \frac{1+(-1)^{n+1}\cdot(-2n-3)}{4} = \\
& \frac{1+(-1)^{n+1}\cdot(-1)(2n+3)}{4} = \\
& \frac{1+(-1)^{(n+1)+1}\cdot(2(n+1)+1)}{4}
\end{aligned}$$

Aufgabe (2)

Induktionsvorraussetzung

$l_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$

Induktionsbasis

$n = 0$

$l_0 = \left(\frac{1+\sqrt{5}}{2}\right)^0 + \left(\frac{1-\sqrt{5}}{2}\right)^0 = 1 + 1 = 2$

$n = 1$

$l_1 = \left(\frac{1+\sqrt{5}}{2}\right)^1 + \left(\frac{1-\sqrt{5}}{2}\right)^1 = \frac{1+\sqrt{5}+1-\sqrt{5}}{2} = \frac{2}{2} = 1$

$n = 2$

$l_2 = \left(\frac{1+\sqrt{5}}{2}\right)^2 + \left(\frac{1-\sqrt{5}}{2}\right)^2 = 1 + \frac{1+\sqrt{5}}{2} + 1 + \frac{1-\sqrt{5}}{2} = 2 + \frac{1+\sqrt{5}+1-\sqrt{5}}{2} = 2 + 1 = 3$

$l_2 = l_1 + l_0 = 1 + 2 = 3$

Induktionsschritt für $n+1$

$$l_{n+1} = l_{(n+1)-1} + l_{(n+1)-2} = l_n + l_{n-1} =$$

$$\left(\frac{1+\sqrt{5}}{2}\right)^n \left(\frac{1-\sqrt{5}}{2}\right)^n + \left(\frac{1+\sqrt{5}}{2}\right)^{n-1} + \left(\frac{1-\sqrt{5}}{2}\right)^{n-1} =$$

$$\left(\frac{1+\sqrt{5}}{2}\right)^n \left(\frac{1-\sqrt{5}}{2}\right)^n + \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n}{\frac{1+\sqrt{5}}{2}} + \frac{\left(\frac{1-\sqrt{5}}{2}\right)^n}{\frac{1-\sqrt{5}}{2}} =$$

$$\frac{\left(\frac{1+\sqrt{5}}{2}\right)^{n+1}}{\frac{1+\sqrt{5}}{2}} + \frac{\left(\frac{1-\sqrt{5}}{2}\right)^{n+1}}{\frac{1-\sqrt{5}}{2}} + \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n}{\frac{1+\sqrt{5}}{2}} + \frac{\left(\frac{1-\sqrt{5}}{2}\right)^n}{\frac{1-\sqrt{5}}{2}} =$$

$$\frac{\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} + \left(\frac{1+\sqrt{5}}{2}\right)^n}{\frac{1+\sqrt{5}}{2}} + \frac{\left(\frac{1-\sqrt{5}}{2}\right)^{n+1} + \left(\frac{1-\sqrt{5}}{2}\right)^n}{\frac{1-\sqrt{5}}{2}} =$$

$$\frac{\left(\frac{1+\sqrt{5}}{2}\right)^n \left(\frac{1+\sqrt{5}}{2}\right) + \left(\frac{1+\sqrt{5}}{2}\right)^n}{\frac{1+\sqrt{5}}{2}} + \frac{\left(\frac{1-\sqrt{5}}{2}\right)^n \left(\frac{1-\sqrt{5}}{2}\right) + \left(\frac{1-\sqrt{5}}{2}\right)^n}{\frac{1-\sqrt{5}}{2}} =$$

$$\left(\frac{1+\sqrt{5}}{2}\right)^n \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n \left(\frac{1-\sqrt{5}}{2}\right)^n =$$

$$\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} + \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}$$

Aufgabe (3)

$(log(n^2))^2$ vor $\sqrt{100n}$ vor n^{lnlnn} vor 2^n

$$
\begin{aligned}
(log(n^2))^2 &\leq c \cdot \sqrt{100n} && f_4 \in O(f_2) \\
(log(n^2))^2 &\leq 10c \cdot \sqrt{n} \\
2 \cdot log(n^2) \cdot \frac{2}{n} &\leq \frac{5c}{\sqrt{n}} && \mathit{Ableitung} \\
\frac{4 \cdot log(n^2)}{n} &\leq \frac{5c}{\sqrt{n}} \\
\frac{8 \cdot log(n)}{n} &\leq \frac{5c}{\sqrt{n}} \\
8 \cdot log(n) &\leq 5c \cdot \sqrt{n} && c = \frac{8}{5} \\
log(n) &\leq \sqrt{n}
\end{aligned}
$$

$$
\begin{aligned}
\sqrt{100n} &\leq c \cdot n^{lnlnn} && f_2 \in O(f_3) \\
10n^{\frac{1}{2}} &\leq c \cdot n^{lnlnn} && c = 10 \\
n^{\frac{1}{2}} &\leq n^{lnlnn} \\
\frac{1}{2} &\leq lnlnn
\end{aligned}
$$

$$
\begin{aligned}
n^{lnlnn} &\leq c \cdot 2^n && f_3 \in O(f_1) \\
ln(n^{lnlnn}) &\leq ln(c \cdot 2^n) && c = 1 \\
ln(n^{lnlnn}) &\leq n \cdot ln(2) \\
\frac{1}{n} + \frac{ln(ln(n))}{n} &\leq ln(2) && \text{Ableitung} \\
1 + ln(ln(n)) &\leq n \cdot ln(n) && \text{Multiplizieren mit n} \\
\frac{1}{n \cdot ln(n)} &\leq ln(2) && \text{Ableitung} \\
1 &\leq n \cdot ln(n) \cdot ln(2) \\
0 &\leq ln(n) \cdot ln(2) + ln(2) && \text{Ableitung} \\
0 &\leq \frac{ln(2)}{n} \\
0 &\leq ln(2)
\end{aligned}
$$

Aufgabe (4)

$$2^{100} \cdot n^2 \leq c \cdot n^2 \qquad 2^{100} \cdot n^2 \in O(n^2)$$
$$\frac{2^{100} \cdot n^2}{n^2} \leq c$$
$$2^{100} \leq c$$

Dies gilt immer, da man lediglich $c \geq 2^{100}$ wählen muss.
$\lim\limits_{n \to \infty} \frac{2^n}{e^n} = \lim\limits_{nto\infty} \left(\frac{2}{e}\right)^n = 0$
Daraus folgt $2^n \in o(e^n)$, folglich ist auch $e^n \in \omega(2^n)$.